$$Y_1=\lambda y_1+\sqrt{(1-\lambda^2)}y_2$$
$$Y_2=-\sqrt{(1-\lambda^2)}y_1+\lambda y_2$$

as the group of the equation

$$(8)\qquad x(1-x)\frac{d^2y}{dx^2}-\frac{x}{2}\frac{dy}{dx}+a^2y=0,\qquad (a=\text{constant})$$

since the relation between a suitably selected pair of fundamental integrals is $y_1^2+y_2^2=1$.*

On the basis of our theorem however, G_r becomes G_1:

$$\begin{cases} Y_1=\lambda y_1+\sqrt{(1-\lambda^2)}y_2, \\ Y_2=-\sqrt{(1-\lambda^2)}y_1+\lambda y_2, \end{cases}\qquad \begin{cases} \bar{Y}_1=\lambda y_1-\sqrt{(1-\lambda^2)}y_2, \\ \bar{Y}_2=\sqrt{(1-\lambda^2)}y_1+\lambda y_2; \end{cases}$$

and the group of rationality of (8) will be G_1 or the identity according to the domain of rationality which is selected. Since (8) is a special case of the hypogeometric equation

$$x(1-x)\frac{d^2y}{dx^2}+[Y-x(1+a+\beta)]\frac{dy}{dx}-a\beta y=0$$

the adjunction of the hypogeometric function would reduce the group of rationality to the identical transformation.

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* Picard, *loc. cit.*

SOME FALLACIES IN TEXT-BOOKS ON ELEMENTARY SOLID GEOMETRY.

By PROF. G. W. GREENWOOD, McKendree College, Lebanon, Ill.

So much has been written on the fallacies encountered in elementary plane geometries in applying to curves theorems on the lengths of broken lines, that it is, perhaps, superfluous to note the more frequent errors of similar character in elementary texts on solid geometry.

For example, in the case of a cylindrical surface, there is in its definition no connection between it and a prismatic surface; and any attempt to *prove* that the surface of the inscribed or circumscribed prism approaches the surface of the cylinder as the number of sides is indefinitely increased, seems fallacious. For no matter how great is the number of sides of the inscribed prism, there is still an *infinite gap* between it and the cylinder in which it is inscribed, and to assume

that a relation between these surfaces, which initially is *entirely absent*, is supplied by multiplying the number of sides of the prism, is to deceive ourselves.

Equally vulnerable seems the corollary commonly attached, that the lateral area of the cylinder is therefore the limit of the lateral area of the inscribed prism when the number of sides is indefinitely increased; for such a corollary presupposes that we have already defined the *area* of a *curved surface* in some other way.

Such unjustifiable assumptions may be avoided by first showing that the area of the inscribed or circumscribed prism has a limit when the number of sides is indefinitely increased, and then *defining* the area of the cylinder as the limit of the area of the inscribed or circumscribed cylinder.

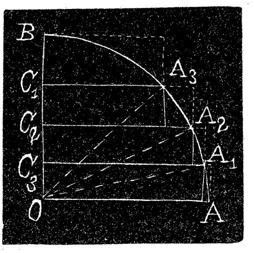
THE VOLUME OF THE SPHERE.

By HÉNRY L. COAR, A. M., University of Illinois.

The solids of revolution, that is, the solids generated by the revolution of a plane figure about an axis, offer many interesting problems to the student of synthetic geometry of three dimensions. The question of obtaining the volumes and total areas of solids, generated by the revolution of rectilinear figures, presents no particular difficulty, but when we attempt to find by purely synthetic means the volumes and superficial area of solids formed by curvilinear figures, the problem is not always simple without making some assumptions regarding limits. A well-known problem of this kind is to find the volume and area of an anchor-ring. A most interesting problem in this line is that of obtaining the volume of a sphere regarded as a solid of revolution. In the following proof, which I have not been able to find published anywhere, no assumptions of any kind regarding the existence of the limits are necessary.

We need consider only the hemisphere, which is generated by the revolution, through 360°, of a quadrant of a circle about a radius.

Let AOB be the quadrant of a circle and let us divide the radius OB into n equal parts. Then construct a set of inscribed and a set of circumscribed rectangles as indicated in the figure.



If now we rotate the complete figure through 360° about the radius OB, the quadrant will generate a hemisphere, while each of the inscribed as well as each of the circumscribed rectangles will generate a right circular cylinder. Let us designate the sum of the volumes of the cylinders generated by the inscribed rectangles by V_1, that of the cylinders generated by the circumscribed rectangles by V_2, and the volume of the hemisphere by V. We will first prove that both V_1 and V_2 approach V as their limit as n increases indefinitely, and